The Epq Model With Emergency Maintenance, Imperfect Products, Backorders And Work In Process Inventory

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Received: 22.03.2015; Accepted: 29.05.2015

Abstract. The Economic Production Quantity (EPQ) is one of the most popular models in inventory control environments. Although this model has been developed influenced by many factors, but in some areas it has received little attention that the examination of the machine failure is one. In this paper, taking into account the assumptions such as the possibility of producing imperfect products with the ability to re-work and without the ability to rework and etc. the EPQ model has been developed in applying the policy of Emergency Maintenance (EM) and a method for achieving optimal size of production batch has been offered. The results of the model sensitivity analysis show that work in process (WIP) inventory holding cost rate while having an inverse relationship with the optimum batch size production, has the greatest impact in determining the amount.

Keywords: Maintenance; Production; Inventoty; Optimization

Introduction

Lot-Sizing economic models for the first time have been studied extensively by Harris (1913). He has published the results of his research in an article entitled 'How many parts to make at once factory?' In which he has introduced his popular model, the Model Economic Order Quantity (EOQ) that lead to create a balance between launching and transportation costs. Five years later, the Economic Production Quantity (EPQ) model was proposed by Taft (1918) that the model is already widely used in the industry. These two models at their rise were based on a set of unrealistic conditions and assumptions which in practice, made them limited their application in industry that researchers began to develop its classic form in order to increase their efficiency.

EPQ is a simple mathematical model of inventory management in a production–stock systems that is being applied as one of the most popular models of Lot-Sizing in management and control of inventories in industry(Osteryoung et al., 1986). This model can be considered as a developed EOQ model obtained with regard to the constant product rate in EOQ model (Rasti et al., 2011).

Regardless of such acceptance, some researchers have questioned its practical applications due to several unrealistic assumptions regarding model input parameters, which are the setup cost, holding cost and demand rate (Jabber et al., 2004). Thus, a number of researchers have been sought to create changes and develop it in order to adopt EPQ more with real situations.

One of the basic assumptions of the classical model of EPQ is that 100% of the items are perfect. However, this assumption may not be valid for most production environments (Eroglu and Ozdemir, 2007). In other words, in EPQ model, it is assumed that a reliable machine produces all items with a constant production rate and in the manufacturing process, no imperfect items have been produced. In some studies the issue of machine failure is ignored.

Basic EPQ models were developed with partial backorders at a fixed percentage rate for the first time by Mak (1987), Zeng (2001) and Pentico et al. (2009). These are all single-product models in which it is assumed that (1) all parameters are clear and consistent in an infinite time horizon; (2) assumptions relating to costs in the EOQ and EPQ models are permanent and common assumptions; (3) the assumption of replenishment is at a finite rate of P.
Wee et al. (2007) developed an optimal inventory model for items with imperfect quality and shortage backordering. Drake et al. (2011) have considered the batch production planning for a two-stage system in which the final product is planned by applying an EPQ model with backorders. They also have presented a batch optimal inventory store strategy for an EPQ model with partial backorders at a constant rate of $\beta$. Hu and Liu (2010) have examined the optimal replenishment policy under permissible delay in payments and allowable backorders in the EPQ model. They have assumed that finite replenishment rate and unit selling price is not necessarily equal to the purchase price. Widyadana and Wee (2010) have also used an algebraic approach to propose an EPQ model in which the imperfect products recycling was done based on two types of recycling policies. Xu (2009) has offered an optimal policy for a problem by taking into account the inventory as dynamic and stochastic and capacity commitment. Basnet and Leung (2005) have generalized the lot-size problem from a single product to multi-product case. However, only a limited number of researchers considered developing multi-product EPQ models along with partial backorders. Taleizadeh et al. (2010) have examined the problem of periodical multi-product production in a single machine with limited capacity; when both conditions of imperfect items produce and partial backorders are established. Hayek and Salameh (2001) have developed an EPQ model for the case in which the percent of imperfect products has a uniform distribution. Principal assumptions used in the model are: Backorders were permitted and all imperfect products are reworked in order to achieve the good quality and the time of rework is also considered in the model. Chiu (2003) has developed the model of Hayek and Salameh (2001) with an accession of the assumption that instead of all the imperfect products, only a part of them are reworked for achieving the good quality and remaining will be sold at sale prices.

When a machine has long been undergoing maintenances, in this situation it is possible that it produces a percentage of imperfect products which these imperfect products must be reworked. Jamal et al. (2004) have examined the issue of product economic batch with respect to the rework process for the single product and obtained the economic batch in the single product. Chiu et al. (2007) have presented a method to determine the optimum time of processing in an EPQ model which is associated with scraps, rework and stochastic machine failures.

In many manufacturing systems, Work In Process inventory (WIP) is one of the components of inventory cost which is not considered usually in formulas of Lot-Sizing. Boucher (1984) computed the lot holding for WIP and developed a model named Group Technology Order Quantity (GTOQ) whose model in fact is the generalization of EOQ. The main difference between Boucher's modeling with other models is his emphasis on WIP holding cost. Based on the experimental results, he indicated that the GTOQ model is useful in the cases where demand is high and (or) the processing time is significant.

Nowadays, according to the major share of the cost of maintenance in the organization production costs and therefore the cost of the final product, applying an appropriate system in this field can reduce the final price of the product and in addition it helps the improvement of product quality and reliability; so in this regard various strategies have emerged that each one according to their characteristics are allocated to apply in the production certain conditions, such as Emergency Maintenance (EM), Preventive Maintenance (PM), Corrective Maintenance (CM), Total Productive Maintenance (TPM); Reliability Centered Maintenance (RCM), Lean Maintenance, Agile Maintenance and etc.

Emergency Maintenance (EM): In this case, after occurring the first failure in the equipment, the equipment will be repaired. So here in order to repair all kind of failures, the maintenance team of organization shall be in addition to being a highly proficient, able to predict the correct tools needed to carry out the necessary repairs.

Alimohamadi et al. (2011) studied a new EPQ model by applying Preventive Maintenance (PM) and by minimization of the total cost per unit time, they presented a method for determining the optimal size of production batch. In this paper, we extend their model by applying EM policy in the model.
Problem Description

Based on Alimohamadi et al. (2011) model, a single-product and single-machine manufacturing process is intended here which produces periodically a product. Raw materials needed for the process is ordered as a batch of $Q$ size and transferred to the workshop. In each cycle of production, a deposit of unprocessed items of $Q$ size will be placed in a container and then it will be machined. Manufacturing process is so that after machining operation on a set of input materials, in addition to produce perfect (or good quality) products, a percentage of imperfect products will also be produced. Completing the machining cycle, all products will be inspected to specify that if they are perfect or imperfect. Perfect products will be transferred to a separate container which the imperfect products will be divided into two groups of with the ability to re-work and without the ability to re-work. Imperfect products with capabilities of rework once again re-enter the machining process and due to performing machining with more precision in second phase, all will be converted into perfect products and will be transferred to the container of perfect products but imperfect products which lack the ability to rework have been transferred to another container to be sold as wastes at the end of the process.

In this model the shortage will be allowed as backorders, in such a way that in any cycle we can have shortage to the permissible value of $a$ and to compensate it at the beginning of the next cycle.

The possibility of discontinuity during the production process is one of the main assumptions in the model. Accordingly, a machine may face some failure in some production cycles to stop working deliberately which in this situation the machine will immediately undergo repairs and the production process will resume back as the original shape.

In this process, work in process inventory is made up of 3 components: raw material, perfect products and wastes that in order to reduce the amount of transportation, pieces will be entered together into containers or go out of them. In each cycle, the demands related to that cycle in addition to the backorders relating to the cycle prior are satisfied by perfect products in warehouse. The aim is achieving an optimal production lot size such that the total cost per unit time is minimized.

Notation

- $Q$: Production lot size per cycle
- $R_D$: Demand rate of perfect products per unit time
- $t_s$: Machine setup time (min)
- $\varphi$: The cost of purchasing one unit of raw pieces
- $\delta$: The cost for producing a unit of product
- $\Gamma$: Average of the work in process inventory
- $t_f$: The time of a cycle if machine being failed
- $t_{nf}$: The time of a cycle in a situation that is not facing machine failure
- $L$: Fixed Set-up cost per cycle
- $a$: Maximum allowable shortage
- $t_m$: Machining time of a product (min)
- $P$: The probability of machine failure during a cycle
- $\beta_1$: Proportion of re-workable imperfect products
- $\beta_2$: Proportion of non-re-workable imperfect products
- $Q_{EM}$: Optimal size of production batch
- $\varepsilon$: Shortage cost rate per unit time
- $t_R$: The average time required repairing the machine failure
- $\sigma$: Rate of maintenance cost on machine (unit of money per unit time)
- $\rho$: Inspection cost of a unit product
- $\tau$: Inventory holding cost rate (unit of money per unit of money per unit time)
- $M_f$: Monetary value of the average work in process inventory
- $S$: The amount of shortage in a cycle
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\[ I_w \] Average inventory at warehouse
\[ C_T \] Average work in process inventory holding cost per unit time
\[ C_S \] Setup cost per unit time
\[ C_P \] Purchase cost per unit time
\[ C_W \] Holding cost of perfect products in warehouse per unit time
\[ C_A \] Shortage cost per unit time
\[ C_M \] Cost of repairing machine failure per unit time
\[ C_P \] Product inspection cost per unit time
\[ TC_f \] Total cost over a cycle per unit time in the situation of machine failure
\[ TC_{nf} \] Total cost over a cycle per unit time in the event of not occurring machine failure
\[ \overline{TC}_{EM} \] Average of total cost over a cycle per unit time.

Assumptions

1. The inspection of products is 100% and the time of inspection is considered to be 0.
2. Parameters such as the rate of product demand, setup time and the proportion of imperfect product are considered to be a fixed value.
3. In each cycle only once there is the possibility of machine failure and process stop that in the event immediately maintenance team shall take action to eliminate it.
4. The machine failure is accidentally and causes the full stops of process and the machine can also be repaired.
5. The time required to repair the damage in an accidental failures occurring in machine during the process follows a normal distribution, and this time is considered equal to the average of normal distribution.

Modeling

Consider the situation that the workshop manager uses EM policy for the machine, considering that failure in machine during the process is an accidental phenomenon, here two states may occur: either the machine is broken during manufacturing process and machining process has been stopped for a while, or without occurring failure, the process finishes, that in the following we will analyze these states.

1. Assume that in a specific cycle of production process, the machine crashes and consequently, the machining process is stopped for \( t_f \). Now we want to investigate the behavior of the work in process inventory. In this situation, Diagrams of work in process inventory position for the raw pieces, perfect products and wastes are in the form of Fig. 1.

![Fig. 1. Work in process inventory including (1) raw materials (2) perfect products and (3) wastage in machine failure state in a certain cycle.](image1)

![Fig. 2. Perfect products in the warehouse in machine failure state in a certain cycle.](image2)
In Fig. 1 (1) first the machine has been launched in time $t_s$ and then raw pieces were machined. Over time the machine suddenly fails in a moment and subsequently the production process has been stopped which the failure has been removed in the $t_R$ and then machining operations was resumed until the quantity of raw pieces is zero. Machining has been done for the first time on all raw pieces and later on the proportion of $\beta_1$ of raw pieces which have the ability to rework (i.e. totally on $Q(1 + \beta_1)$ unit raw pieces), so the quantity of perfect products at the end of a cycle has reached to $Q(1 - \beta_2)$.

Fig. 1 (2) and Fig. 1 (3) respectively are related to perfect products and wastes, that have an increasing rate by starting machining process and at the end of a sequence they reached to values $Q(1 - \beta_2)$ and $\beta_2 Q$.

Also in this situation, the diagram for inventory status for perfect products in the warehouse is shown in Fig.2 (In order to supply the customers demand and meeting backorders). In Fig. 2 at the beginning of each cycle the perfect products in warehouse are consumed with the rate of demand $R$ so:

$$\frac{C_L}{t_f}$$ (3)

**Setup cost per unit time**

Every product is under investigation just once because by conducting the inspections one time, it will be determined that the product is perfect or it is imperfect and will need to re-work or it is imperfect and it will be wastes. Therefore:

$$C_p = \frac{\rho Q}{t_f}$$ (4)

**Average work in process inventory holding cost per unit time**

Average work in process inventory per unit time for raw pieces, perfect products and wastes are calculated as follows:

$$I = \frac{\frac{1}{2}Q(1 + \beta_1)}{t_f} + \frac{\frac{1}{2}Q(1 - \beta_2)(1 + \beta_1)}{t_f} + \frac{1}{2}Q(1 - \beta_2)$$ (5)

So monetary value of the average work in process inventory per unit time is equal to:

$$M_T = \frac{\frac{1}{2}Q(1 + \beta_1)}{t_f} + \frac{\frac{1}{2}Q(1 - \beta_2)(1 + \beta_1)}{t_f} + \frac{1}{2}Q(1 - \beta_2)$$ (6)

As a result, Average work in process inventory holding cost per unit time is equal to:

$$C_T = \frac{\frac{1}{2}C_T(\delta + \phi)(1 + \beta_1)Q^2}{t_f}$$ (7)

**Holding cost of perfect products in warehouse per unit time**

The average inventory at warehouse is equal to: (According to Fig. 2.)

$$I_w = \frac{(1 - \beta_2)Q^2}{2R_D}$$ (8)

So:
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\[ C_M = \frac{\tau \sigma (1-\beta)^2 a^2}{2R_D} \]  
\[ (9) \]

**Shortage cost per unit time**

According to Fig.2 the amount of shortage in a cycle is equal to:

\[ S = \frac{a^2}{2R_D} + t_R a \]  
\[ (10) \]

Therefore:

\[ C_s = \frac{t \left( \frac{a^2}{2R_D} + t_R a \right)}{t_f} \]  
\[ (11) \]

**Purchases cost per unit time**

In any cycle, an order will be purchased as \( Q \) unit, so:

\[ C_p = \frac{\varphi Q}{t_f} \]  
\[ (12) \]

**Cost of repairing machine failure per unit time**

The cost of failure repair operations on the machine per unit time is equal to:

\[ C_M = \frac{\sigma L_R}{t_f} \]  
\[ (13) \]

So the total cost over a cycle per unit time is equals to:

\[ TC_f = L \frac{\sigma Q}{t_f} + \frac{\varphi Q}{t_f} + \frac{1}{2} \left( \delta + \varphi \right) \left( t_m (1 + \beta_1) Q^2 + t_R Q \right) + \frac{\tau \sigma (1-\beta)^2 a^2}{2R_D} + \frac{\epsilon \left( \frac{a^2}{2R_D} + t_R a \right)}{t_f} + \frac{\varphi Q}{t_f} + \frac{\sigma L_R}{t_f} \]  
\[ (14) \]

2. Consider now the case in which the machine at a particular period without being faced with failure finishes the machining process on the raw pieces. Diagrams of work in process inventory position for the raw pieces, perfect products and wastes are in the form of Fig. 3. Also, the inventory position diagram for perfect products in warehouse is in the form of Fig. 4.

![Fig. 3. Work in process inventory including (1) raw materials (2) perfect products and (3) wastage in not machine failure state in a certain cycle.](image1)

![Fig. 4. Perfect products in the warehouse in not machine failure state in a certain cycle.](image2)

The time of the cycle is the sum of the setup time and processing time:

\[ t_{nf} = t_s + t_m (1 + \beta_1) Q \]  
\[ (15) \]

The total cost is:

\[ TC_{nf} = C_L + C_p + C_r + C_w + C_a + C_p \]  
\[ (16) \]
Setup cost per unit time:
\[ C_L = \frac{L}{t_{nf}} \]  

(17)

Product inspection cost per unit time
\[ C_p = \frac{\rho Q}{t_{nf}} \]  

(18)

Average work in process inventory holding cost per unit time
\[ \bar{I} = \frac{1}{2}Q(t_{m}Q(1+\beta_1))^{\frac{t_{nf}}{t_{nf}}} + \frac{1}{2}Q(1-\beta_2)(t_{m}Q(1+\beta_1))^{\frac{t_{nf}}{t_{nf}}} + \frac{1}{2}\beta_2(t_{m}Q(1+\beta_1))^{\frac{t_{nf}}{t_{nf}}} \]  

(19)

\[ M_f = \frac{1}{2}Q(t_{m}Q(1+\beta_1))^{\frac{t_{nf}}{t_{nf}}} + \frac{1}{2}Q(1-\beta_2)(t_{m}Q(1+\beta_1))^{\frac{t_{nf}}{t_{nf}}} + \frac{1}{2}\beta_2(t_{m}Q(1+\beta_1))^{\frac{t_{nf}}{t_{nf}}} \]  

(20)

\[ C_T = \frac{1}{2}(\delta+\varphi)\tau_{m}(1+\beta_1)Q^2 \]  

(21)

Holding cost of perfect products in warehouse per unit time
\[ C_w = \frac{\tau_0Q^2(1-\beta_2)^2}{2R_D} \]  

(22)

Shortage cost per unit time
\[ S = \frac{a^2}{2R_D} \]  

(23)

\[ C_a = \frac{e^{2R_D}}{t_{nf}} \]  

(24)

Purchases cost per unit time
\[ C_P = \frac{\varphi Q}{t_{nf}} \]  

(25)

So the total cost over a cycle per unit time in the event of not occurring machine failure is:
\[ TC_{nf} = \frac{L}{t_{nf}} + \frac{\varphi Q}{t_{nf}} + \frac{1}{2}(\delta+\varphi)\tau_{m}(1+\beta_1)Q^2 + \frac{\tau_0(1-\beta_2)^2Q^2}{2R_D} + \frac{e^{2R_D}}{t_{nf}} + \frac{\rho Q}{t_{nf}} \]  

(26)

Suppose that the probability of machine failure during machining is equal to \( P \). (so the possibility of non-occurrence of failure in machine is \( (1-P) \)) so the average total cost over a cycle per unit time is equal to:
\[ \overline{TUC}_{EM} = P \times TC_f + (1-P) \times TC_{nf} = \]  

\[ P \times \left[ \frac{L}{t_f} + \frac{\rho Q}{t_f} + \frac{1}{2}(\delta+\varphi)\tau_{m}(1+\beta_1)Q^2 + \frac{\tau_0(1-\beta_2)^2Q^2}{2R_D} + \frac{e^{2R_D}}{t_f} \right] + \]  

\[ (1-P) \times \left[ \frac{L}{t_{nf}} + \frac{\rho Q}{t_{nf}} + \frac{1}{2}(\delta+\varphi)\tau_{m}(1+\beta_1)Q^2 + \frac{\tau_0(1-\beta_2)^2Q^2}{2R_D} + \frac{e^{2R_D}}{t_{nf}} \right] \]  

(27)

Function \( \overline{TUC}_{EM} \) is a curve depending on the variable of product batch size \( Q \) and because it is the cost type, it needs to be minimized. by derivation this function with respect to \( Q \) we will reach a deficit means that the numerator is a polynomial of degree 4, that by being equal to zero the first derivative of function \( \overline{TUC}_{EM} \), we must calculate the 4 degrees equation roots of the numerator, but since there is no systematic methods for reaching the roots of the quadratic equation 4, we cannot achieve a general formula for calculating the optimal value of \( Q \), so compelling have to use numerical methods that is, we should insert the parameters numerical values in the function \( \overline{TUC}_{EM} \) and by applying mathematical software, such as Maple, calculate the roots of the first derivative of the function. Since the term in the numerator the first derivative of the function \( \overline{TUC}_{EM} \) is degree 4, so after finding the roots of the first derivative of this function, we are faced with four different answers that two answers are mixed type, and one is a real negative and finally a real positive answer. So the real
positive answer is the real value of \(Q\) when the second derivative of the function \(TU_CEM\) is positive for it, or in other words, the cost function is a convex function.

**Numerical example**

The following numerical example illustrates how to use the results of the model presented.

1. A workshop produces a certain product using a single machine. The raw pieces needed are transferred from other workshops as a batch and with the price of each unit 9$. Each raw piece is machined at the time of 6 minutes and cost 11$ and are inspected with a fee of 17$. In this process, 15% of the parts are reworked, but 10% of them become wastes. Suppose that the shortage as backorders up to 12 units per cycle is allowed. Inventory holding cost per unit time is 12% and the rate of facing with shortage is 50$ per unit time. Demand rate is 40 units per unit time, and also, the probability of a machine faces a failure in a cycle is 0.35. If the machine setup time takes 5 minutes and fixed cost to setup it is 200$ and the manager wants to use the EM policy, in addition, cost rate of maintenance of machine is 25$ per unit time, determine the optimal size of product batch. Suppose that the time required to repair the machine failure has a normal distribution of mean 24 minutes.

Solution: Data include:

- Machine faces a failure in a cycle is 0.35.
- Demand rate is 40 units per unit time.
- Fixed cost to setup is 200$.
- Cost rate of maintenance of machine is 25$ per unit time.

Now, the value of \(T_C\) and \(T_C_{nf}\) is equal to:

\[
T_C = \frac{8.29Q^2+54.8Q+15290}{6.9Q+29} \\
T_C_{nf} = \frac{8.29Q^2+26Q+290}{6.9Q+5}
\]

So:

\[
TU_CEM = 0.35\times \frac{8.29Q^2+54.8Q+15290}{6.9Q+29} + 0.65\times \frac{8.29Q^2+26Q+290}{6.9Q+5} = \frac{29Q^2+19.18Q+5351.5}{6.9Q+29} + \frac{539Q^2+16.9Q+188.5}{6.9Q+5}
\]

Using the Maple software, positive root of the first derivative of the function \(TU_CEM\) with respect to \(Q\) is given by:

\[
\frac{dTU_CEM(Q)}{dQ} = 0 \Rightarrow Q = 21.7
\]

Reusing the Maple, the second derivative of the function \(TU_CEM\) at the point \(Q = 21.7\) are equal to:

\[
\frac{d^2TU_CEM(Q)}{dQ^2} = 0.09 > 0
\]

So, due to being positive the second derivative of function \(TU_CEM\) at the point \(Q\), the optimal size of production batch is equal to:

\[Q = 21.7 \Rightarrow Q_{EM} = 22\]

**Sensitivity analysis**

In this section various numerical examples are provided to perform sensitivity analysis on model.

1. Suppose that numerical values of other model parameters are fixed and as follows:

- Machine faces a failure in a cycle is 0.35.
- Demand rate is 40 units per unit time.
- Fixed cost to setup is 200$.
- Cost rate of maintenance of machine is 25$ per unit time.

In this case we want to examine the behavior of \(Q_{EM}\) toward the change of the five key parameters \(t_R, a, \tau, \beta_1\) and \(P\). By fixing four of these parameters, the fifth parameter is changed. These results are expressed in Table 1. Note that in all cases the second derivative of the function \(TU_CEM\) is positive at the optimum \(Q\).
Table 1. Results of solving model by Maple software

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<th>$a$</th>
<th>$\tau(S/min)$</th>
<th>$\beta_1$</th>
<th>$P$</th>
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Behavior of $Q_{EM}$ to changes in certain parameters of (Table 1) came in the Fig. 5-7. As it implies, $Q_{EM}$ has a direct relationship with $t_R$ and $a$ parameters, but it has an inverse relationship with $\tau$.

![Fig. 5. Behavior chart of $Q_{EM}$ based on changing $t_R$.](image1)

![Fig. 6. Behavior chart of $Q_{EM}$ based on changing $\tau$.](image2)
CONCLUSIONS AND RESULTS

In this study, using various numerical examples, the application of the results of model in real problems was explained, while the sensitivity level of optimal values of production to some parameters has been investigated.

As the results show, in the case of EM policy (with respect to a certain probability of machine failure), work in process inventory holding cost rate in addition to having an inverse relation with the optimal size of production batch, it has the greatest amount of influence on the determination of the optimal size of production batch. In justifying this issue it can be said that in every cycle in addition to have 3 types of work in process in inventory, we are obliged to pay the cost for holding of perfect products in the warehouse, so the share of this factor in the total cost is more than any other factors; meanwhile the influence of this factor in determining the optimal size of batch production for values less than 10% is more than other values.

REFERENCES


