Modeling And Solving A Pricing Problem Considering Substitutable Products In A Dual Supply Chain With Internet And Traditional Distribution Channels

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Abstract. This paper studies the pricing problem in a supply chain consisting of two manufacturers who whether sell their substitutable products to an independent retailer or sell them directly to consumers through their own Internet channels. Channel and brand competition is considered for each product whose demand function is stochastic and linear. The objective function is to maximize the manufacturers and the retailer profits. Three decentralized pricing policy are developed and the corresponding analytical equilibrium solutions are obtained using the game-theoretic approach for Nash and Stackelberg games. Numerical examples are presented to study the effectiveness of each policy. The results indicate that brand loyalty enhancement is profitable for both the manufacturers and the retailer. Furthermore, a limited increase in service value may decrease the threat of the direct Internet channel for the retailer while increasing the manufacturers’ profit.

Keywords: Dual supply chain, pricing, game theory, substitutable products, Internet distribution channel

INTRODUCTION

A number of industrial and governmental investigations indicate an attractive growing rate within the course of Internet commerce. Manufacturers are increasingly selling their products through direct online channels, in addition to the traditional retail channels (Tsay and Agrawal, 2004). According to Tedeschi (2000), around 42% of top suppliers (within different industries) such as IBM, Hewlett–Packard, Nike, Pioneer Electronics, Mattel, Estee Lauder, the former Compaq, Dell, and Cisco System are directly selling to consumers through their online channels. Recognizing the potential benefits of an Internet channel, many manufacturers have shown interest in establishing Internet channels to complement the traditional retailing channels (Hua, Wang, and Cheng, 2010). In this dual supply chain, consumers have an option to choose the channel better suited to their needs. As Kumar and Ruan (2006) declared, other factors such as service level and brand contribute to consumer needs. Over the last few years, dual supply chains have increased product variety by differentiating some product attributes such as technology, appearance and color in order to remain competitive. However, some of these products may be substitutable because of no distinct differentiation; Kumar and Ruan (2006) showed that the degrees of brand and channel loyalty are influenced by factors such as retail price and manufacturer’s decision to complement the traditional channel with an Internet one. Kurata, Yao, and Liu (2007) analyzed channel pricing in multiple distribution channels under competition between a national and a store brand, where the national brand could distribute through both direct and indirect channels, while the store brand could only do that through indirect channels. Knowing that supply chains consist of several decentralized firms with contributions to the profit

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of the whole supply chain, the importance of reasonable pricing strategies is well-recognized for supply chains. The game theory can be a suitable tool to effectively model and analyze the decisions on pricing in such supply chains. Researchers in supply chain management use the game theory in order to analyze and make pricing decisions for complex multi-agent supply chain systems. Considerable volume of researchers working on pricing strategies within supply chains is an indication of the importance of such an area. The major factors differentiating the researches are product status, supply chain structure and demand type. Table 1 illustrates some recent papers on pricing strategies within supply chain where in the game theory approach is followed.

Table 1. Some of recent papers on pricing strategies within supply chain based on the game theory.

<table>
<thead>
<tr>
<th>no</th>
<th>Paper</th>
<th>Product status</th>
<th>Supply chain structure</th>
<th>Demand type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cai, Zhang, and Michael 2009</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>deterministic</td>
</tr>
<tr>
<td>2</td>
<td>Wei and Zhao 2011</td>
<td>single</td>
<td>one manufacturer + two retailer</td>
<td>fuzzy</td>
</tr>
<tr>
<td>3</td>
<td>Bin, Guangye, and Can 2012</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>deterministic</td>
</tr>
<tr>
<td>4</td>
<td>Bin, Rong, Shuai, and Xindi 2012</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>deterministic</td>
</tr>
<tr>
<td>5</td>
<td>Zhao, Tang, and Wei 2012</td>
<td>substitutable</td>
<td>one manufacturer + two competitive retailers</td>
<td>fuzzy</td>
</tr>
<tr>
<td>6</td>
<td>Zhao and Wei 2012</td>
<td>substitutable</td>
<td>one manufacturer + two retailers</td>
<td>fuzzy</td>
</tr>
<tr>
<td>7</td>
<td>Yongjian, LeiXu , and Dahui 2013</td>
<td>single</td>
<td>one direct distributor who also acts as a manufacturer</td>
<td>deterministic</td>
</tr>
<tr>
<td>8</td>
<td>J. Ma and A. Ma 2013</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>stochastic</td>
</tr>
<tr>
<td>9</td>
<td>Zhiqiang, Fanfan, and Xiangyang 2014</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>deterministic</td>
</tr>
<tr>
<td>10</td>
<td>Jie Tao and Zhao 2014</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>deterministic</td>
</tr>
<tr>
<td>11</td>
<td>Wang and Zhao 2014</td>
<td>single</td>
<td>one manufacturer + one retailer</td>
<td>deterministic</td>
</tr>
</tbody>
</table>

To the best of our knowledge, no research has yet studied the pricing problem within a dual supply chain structure with substitutable products produced by two different manufacturers and two distribution channels. Figure 1 represents the dual supply chain structure of the problem under study.

Figure 1. The dual supply chain structure under study.

In this research, end consumer demand is assumed to be stochastic. We have considered two manufacturers of substitutable products selling their products through both Internet and traditional retailing channels. The major interest is to investigate the pricing policies of the manufacturers in direct and indirect (i.e. retailing) channels as well as those of the retailer for the

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two substitutable products in the stochastic environment. On the other hand, the main contribution of this article is to solve the pricing problem by considering the competition between Internet and traditional channels for both the manufacturers and the retailer while assuming uncertainties on the end consumer demand. This paper studies three scenarios for the manufacturers and the retailer: a Nash game, a Stackelberg game with manufacturers as leaders and a Stackelberg game with retailer as the leader. Using the game theory’s analytical approach, the corresponding optimal pricing strategies as well as the existence and uniqueness conditions of the equilibrium solutions are derived. We will also study and analyze the effects of consumer brand loyalty, channel loyalty and service value on profits and prices.

Considering Figure 1, Manufacturer 1 gives Product 1 to the retailer at the wholesale price \( w_1 \), while it simultaneously gives the same product to consumers directly through its own Internet channel at the retail price \( p_1 \). The retailer gives Product 1 to the consumer at the retail price \( p_1 \). On the other hand, the retailer also gives a substitutable product (i.e. Product 2) to the consumer at the retail price \( p_2 \), Produced by Manufacturer 2. Product 2 is distributed to the retailer at the wholesale price \( w_2 \). It is also given to the consumers directly through an Internet channel at the retail price \( p_2 \). Since the wholesale price is usually determined as a part of a long-term contract between the manufacturer and the retailer, we assume that wholesale prices \( w_1 \) and \( w_2 \) are constant, as is assumed in Dumrongsiri et al. (2008) and Kurata, Yao, and Liu (2007). In the proposed model, Manufacturer 1 decides on the retail price \( p_1 \), Manufacturer 2 decides on the retail price \( p_2 \) and the independent retailer decides on the prices \( p_1 \) and \( p_2 \) in order to maximize their respective profits. As compared with the Internet channel, the retailer can provide better services such as live advertisement, sales explanation, immediate response, attractive environment and personal interaction to consumers through its traditional channel. A linear demand function is assumed to formulate consumer demand. The demand for a product on a channel depends on its price, service value and brand and channel loyalties. Brand loyalty is represented by cross-brand price sensitivity parameter which is defined as the measure of the responsiveness of each product’s demand to its competitive product’s price. Channel loyalty is represented by cross-channel price sensitivity parameter which is defined as the measure of the responsiveness of each product’s demand in a given channel to its price in other distribution channels. We also use a self-price sensitivity parameter in the demand function which is defined as the measure of the responsiveness of each product’s demand to its own price.

The rest of this paper is organized as follows. Section 2 gives the notation. In Section 3, we develop the model and give the uniqueness conditions for equilibrium solutions of the corresponding Nash and Stackelberg games. Section 4 gives a numerical study in order to analyze the effects of brand loyalty, channel loyalty and service value on the profits of the manufacturers and the retailer. Finally, in Section 5, we give conclusions and ideas for future researches.

**Notation**

The following notations are employed in problem formulation.

\( \alpha_0 \): The cross-channel price sensitivity between Product 2 sold through the Internet channel and the same product sold through the traditional channel

\( \alpha_1 \): The cross-brand price sensitivity between Product 2 sold through the Internet channel and Product 1 sold through the traditional channel

\( \alpha_2 \): The cross-brand price sensitivity between Products 1 and 2 both sold through the Internet channel

\( \beta_0 \): The cross-channel price sensitivity between Product 1 sold through the Internet channel and the same product sold through the traditional channel

\( \beta_1 \): The cross-brand price sensitivity between Product 1 sold through the Internet channel and Product 2 sold through the traditional channel
\( \beta_2 \): The cross-brand price sensitivity between Products 1 and 2 both sold through the traditional channel

\( D_0 \): The demand function for Product 1 when sold through the Internet channel

\( D_1 \): The demand function for Product 1 when sold through the traditional channel

\( D_2 \): The demand function for Product 2 when sold through the traditional channel

\( D_3 \): The demand function for Product 2 when sold through the Internet channel

\( a_i \): The potential demand of \( D_i \)

\( b_i \): The self-price sensitivity of demand \( D_i \)

\( X_i \): Stochastic variable of demand \( D_i \)

\( x_i \): A continuous random variable with CDF \( F_i(.) \) over interval \([x_i, x_i] \) assuming \( E(x_i) = 1 \)

\( q_i \): The order quantity of \( D_i \)

\( Y_i \): The deliverable quantity of the order quantity \( q_i \)

\( y_i \): A continuous random variable with CDF \( F_i(.) \) over the interval \([y_i, y_i] \) assuming \( E(y_i) = 1 \)

\( c_i \): The distribution cost per unit of Product \( i \)

\( s_i \): The salvage value per unit of unsold of Product \( i \) in the inventory

\( t_i \): The shortage cost per unit of Product \( i \)

\( m_i \): The salvage value for each unit of unallocated inventory when the allocated capacity exceeds order quantity \( q_i \)

\( z_i \): The stock factors for \( D_i \)

\( W_i \): The unit wholesale price of product \( i \)

MODEL DEVELOPMENT

Based on the supply chain structure given in Figure 1 and the operations of the supply chain given in section 1 while considering a single period problem, we developed the pricing model. Considering the given definitions for the cross-brand price sensitivities between the two products sold through different channels (i.e. \( \alpha_1, \beta_1 \)), for the cross-brand price sensitivities between the two products sold through the same channels (i.e. \( \alpha_2 \) and \( \beta_2 \)) and for the cross-channel price sensitivities of each product sold through different channels (i.e. \( \alpha_0 \) and \( \beta_0 \)), one can establish the demand functions for each product sold through each channel. As mentioned earlier, compared with the Internet channel, the traditional channel can provide better services. It is clear that the service values \( v_0 \) and \( v_3 \) added to Products 1 and 2, respectively, sold through the Internet channels are negligible, while the service values \( v_1 \) and \( v_2 \) added to Products 1 and 2, respectively, sold through the traditional channel are considerable. It is reasonable that \( p_i > v_i \). Kurata, Yao, and Liu (2007) showed that marketing actions, such as promotion, advertising campaigns and better product presentation could cause more frequent brand switching, leading to an increase in \( \alpha_1, \alpha_2, \beta_1 \) and \( \beta_2 \). Furthermore, decreasing channel loyalty causes more frequent channel switching leading to an increase in \( \alpha_0 \) and \( \beta_0 \). Linear demand functions were used to formulate consumer demand as they are widely used in similar researches such as Dai et al. (2005), Yao and Liu(2005), Kurata, Yao, and Liu (2007) and Yao, Yue, and Liu(2008). \( D_0, D_1, D_2, D_3 \) can be obtained by Equations (1), (2), (3), (4), respectively.

\[
\begin{align*}
D_0 &= a_0 - b_0 p_0 + \beta_0 (p_1 - v_1) + \beta_1 (p_2 - v_2) + \alpha_0 p_3 \\
D_1 &= a_1 - b_1 (p_1 - v_1) + \beta_0 p_0 + \beta_1 (p_2 - v_2) + \alpha_1 p_3 \\
D_2 &= a_2 - b_2 (p_2 - v_2) + \beta_1 p_0 + \beta_2 (p_1 - v_1) + \alpha_2 p_3 \\
D_3 &= a_3 - b_3 p_3 + \alpha_0 (p_2 - v_2) + \alpha_1 (p_1 - v_1) + \alpha_2 p_0
\end{align*}
\]

(Eq. 1)

(Eq. 2)

(Eq. 3)

(Eq. 4)

We may consider a random behavior for demand as well as its dependence to the price. For this purpose, we utilized Hsiehand Wu (2009) multiplicative form where stochastic variable of
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demand for Product $i$, $X_i$, can be stated as $D_i \times x_i$. Correspondingly, the profit functions of the retailer and the manufacturers are influenced by the stochastic behavior of demand. Retailer may have different reactions toward risk. If, $z_i$ denotes the stock factors for $D_i$ and the ordering quantity $q_i$, is defined as $D_i \times z_i$, then a retailer with lower value of $z_i$ is considered to be risk-averse, while a retailer with higher value of $z_i$ is considered to be risk-prone. However, the supplied quantities of Products 1 and 2 by Manufacturers 1 and 2 are also uncertain. This uncertainty is also considered in a multiplicative form (i.e. $Y_i = q_i \times y_i$), in which $Y_i$ represents the delivered quantity of the order quantity $q_i$. The profit function for Manufacturer 1, $\pi_{m1}$, is the sum of the Internet channel profit, $\pi_{m1(t)}$, and the traditional channel profit, $\pi_{m1(T)}$, as in Equation (5).

$$\pi_{m1(p_0,p_1,p_2,p_3)} = \pi_{m1(t)}(p_0,p_1,p_2,p_3) + \pi_{m1(T)}(p_0,p_1,p_2,p_3) \quad \text{(Eq. 5)}$$

Where the Internet channel profit $\pi_{m1(t)}$ can be stated as in Equation (6).

$$\pi_{m1(t)}(p_0,p_1,p_2,p_3)=E[p_0 \min[X_0,\min(q_0,Y_0)] - c0\min(q_0,Y_0) + m0\max(Y_0-q_0,0) + s0\max[\min(q_0,Y_0)-X_0,0]-\pi]_0 - \pi\max[X_0-\min(q_0,Y_0),0]) \quad \text{(Eq. 6)}$$

In Equation (6), note that the term $\min(q_0,Y_0)$ represents the delivered order quantity in the Internet channel while the term $\min[X_0,\min(q_0,Y_0)]$ represents the sold quantity in the Internet channel, both for Product 1; therefore $E[p_0 \min[X_0,\min(q_0,Y_0)]]$ gives the expected revenue obtained via the Internet channel. The term $E[c0\min(q_0,Y_0)]$ represents the expected distribution cost of Product 1 through the Internet channel. The term $\max(Y_0-q_0,0)$ represents the unallocated inventory of Product 1 when the allocated capacity to the Internet channel exceeds the order $q_0$, and $E[m0\max(Y_0-q_0,0)]$ gives the expected salvage value for unallocated inventory of Product 1 within the Internet channel. The term $\max[\min(q_0,Y_0)-X_0,0]$ represents Product 1’s unsold inventory in the Internet channel and $E[s0\max[\min(q_0,Y_0)-X_0,0])$ gives its expected salvage value, $\max[X_0-\min(q_0,Y_0),0]$ represents the shortage quantity of Product 1 in the Internet channel and $E[t0\max[X_0-\min(q_0,Y_0),0]]$ gives the expected shortage cost.

The traditional channel profit $\pi_{m1(T)}$ can be stated as in Equation (7).

$$\pi_{m1(T)}(p_0,p_1,p_2,p_3)=E[(w_1-c1)\min(q_1,Y_1)+m1\max(Y_1-q_1,0)] \quad \text{(Eq. 7)}$$

In Equation (7), note that the term $\min(q_1,Y_1)$ represents Product 1’s delivered order quantity from Manufacturer 1 to the retailer, while $w_1-c1$ is the net profit per unit of Product 1 sold to the retailer. Seemingly, the term $E[(w_1-c1)\min(q_1,Y_1)]$ gives the expected net profit of Product 1 sold via the traditional channel while $E[m1\max(Y_1-q_1,0)]$ gives the expected salvage value of unallocated inventory when the allocated capacity exceeds the order quantity $q_1$.

Similarly, Manufacturer 2’s profit function $\pi_{m2}$ can be formulated as in Equation (8).

$$\pi_{m2}(p_0,p_1,p_2,p_3)=\pi_{m2(t)}(p_0,p_1,p_2,p_3) + \pi_{m2(T)}(p_0,p_1,p_2,p_3) \quad \text{(Eq. 8)}$$

where $\pi_{m2(t)}(p_0,p_1,p_2,p_3)$ and $\pi_{m2(T)}(p_0,p_1,p_2,p_3)$ can be expressed as Equations (9) and (10), respectively.

$$\pi_{m2(t)}(p_0,p_1,p_2,p_3)=E[p_0 \min[X_0,\min(q_3,Y_3)] - c_3\min(q_3,Y_3) + m_3\max(Y_3-q_3,0) + s_3\max[\min(q_3,Y_3)-X_3,0]-\pi]_0 - \pi\max[X_3-\min(q_3,Y_3),0]) \quad \text{(Eq. 9)}$$

And $\pi_{m2(T)}(p_0,p_1,p_2,p_3)$ can be stated as in Equation (10).

$$\pi_{m2(T)}(p_0,p_1,p_2,p_3)=E[(w_2-c_2)\min(q_2,Y_2)+m_2\max(Y_2-q_2,0)] \quad \text{(Eq. 10)}$$
The retailer’s profit $\pi_R$ which is the sum of the two product’s profits can be obtained by Equation (11), where $\pi_{R(i)}$ represents the profit function for Product $i$ and can be stated as in Equation (12).

$$\pi_R(p_0,p_1,p_2,p_3)=\sum_{i=1}^{3} \pi_{R(i)}(p_0,p_1,p_2,p_3)$$  \hspace{1cm} (Eq. 11)

$$\pi_{R(i)}(p_0,p_1,p_2,p_3)=E\{p_i\min[X_i,\min(q_i,Y_i)]-[W_i+c(v_i)]\min(q_i,Y_i)+s_i\max[\min(q_i,Y_i)-X_i,0]-t_i\max[X_i-\min(q_i,Y_i),0]\}$$  \hspace{1cm} (Eq. 12)

Equation (12) consists of a revenue term $p_i\min[X_i,\min(q_i,Y_i)]$ as well as a term representing the sum of wholesale price and service costs $[W_i+c(v_i)]\min(q_i,Y_i)$ and also salvage and shortage costs related to Product $i$ sold through the traditional retailer channel. In Equation (12), note that the service cost function $c(v_i)$ is utilized in order to represent the relationship between $v_i$ and its related service cost. One usual and practical form of the service cost function is $c(v_i)=\eta_i v_i^2/2$, in which $\eta_i (\eta_i>0)$ represents the service cost parameter according to in Yao and Liu (2005) and Yao, Yue, and Liu (2008). We used the addressed term for $c(v_i)$ in numerical examples.

Equations (9)-(11) can be rewritten in the form of Equations (13)-(15), respectively.

$$\pi_{m1}=[p_0\theta_0-C_0\theta_0+m_0(1-\lambda_0)+s_0(\lambda_0-\theta_0)-t_0(k_0-\lambda_0)]z_0D_0+[max[\min(q_1,Y_1)]]-t_i\max[X_i-\min(q_i,Y_i),0]$$  \hspace{1cm} (Eq. 13)

$$\pi_{m2}=[p_0\theta_0-C_0\theta_0+m_0(1-\lambda_0)+s_0(\lambda_0-\theta_0)-t_0(k_0-\lambda_0)]z_0D_0+[max[\min(q_1,Y_1)]]-t_i\max[X_i-\min(q_i,Y_i),0]$$  \hspace{1cm} (Eq. 14)

$$\pi_{R(i)}(p_0,p_1,p_2,p_3)=\sum_{i=1}^{3} \{p_i\theta_i[(1-c(v_i))]z_iD_i+s_i(\lambda_0-\theta_0)-t_i(1-\lambda_0)]z_iD_i$$  \hspace{1cm} (Eq. 15)

Where $\theta_i$, $k_i$, and $\lambda_i$ are defined in Equations (16)-(18).

$$\theta_i=E[\min\{x/z_i, \min\{1,y_i\}\}] \hspace{1cm} i=1,2 \hspace{1cm} (Eq. 16)$$

$$k_i=E[\max\{x/z_i, \min\{1,y_i\}\}] \hspace{1cm} i=1,2 \hspace{1cm} (Eq. 17)$$

$$\lambda_i=E[\min\{1,y_i\}] \hspace{1cm} i=1,2 \hspace{1cm} (Eq. 18)$$

$\theta_i$, $k_i$, $\lambda_i$ can be interpreted as the average sales volume of Product $i$, average demand volume of Product $i$, and average quantity delivered per unit of the order quantity $q_i$, respectively.

Based on the demand and profit functions given in Equations (1)–(15), the following assumptions are made:

**Assumption 1:** $b_0>b_0+b_1+a_2, b_1>b_0+b_2+a_3, b_2>b_1+b_3+a_0, b_3>b_0+a_1+a_2$

Assumption 1 states that in general the influence of $p_i$ ($i=0,1,2,3$) on $D_i$ will be greater than the aggregate influence of the other prices and it states each firm comes across a decrease in its sales volume if all firms simultaneously increase their prices by the same amount, a condition which appears to be satisfied in most industries as Bernstein and Federgruen (2004) and Dai et al. (2005), declared.

**Assumption 2:** $\pi_{m1}, \pi_{m2}, \pi_{R1}, \pi_{R2} \geq 0$

The participation of the retailer and Manufacturers 1 and 2 in the business is guaranteed by Assumption 2.

Based on the given assumptions and results, the pricing problem can be stated as in Equations (19)-(22).

Max $\pi_{m1}, \pi_{m2}, \pi_{R1}, \pi_{R2}$ \hspace{1cm} (Eq. 19)

S.t:

$$p_0, p_1, p_2, p_3 \geq 0$$ \hspace{1cm} (Eq. 20)

$$p_1 \geq w_1$$ \hspace{1cm} (Eq. 21)

$$p_2 \geq w_2$$ \hspace{1cm} (Eq. 22)

To study the effect of bargaining powers of the manufacturers and the retailer in the supply chain, some pricing schemes are given in three different scenarios: Nash game, Stackelberg game when the retailer is the leader, and Stackelberg game when the Manufacturers are the leaders.
Pricing policy under Nash game

In the Nash game, all elements of the system are assumed to be of equal bargaining powers. For the current model, manufacturers and the retailer simultaneously decide on their prices so as to maximize their own profits. The outcome of the game is not clear when an equilibrium solution does not exist. However, the decision maker can characterize the optimal solution without vagueness and ambiguity if an equilibrium solution exists. Trying to recognize the existence and uniqueness of an equilibrium solution, Cachon and Netessine (2004) provided useful theoretical tools.

This paper uses the following results proposed by Topkis (1979), Topkis (1998) and Milgrom and Roberts (1990):

**Definition 1 (Cachon and Netessine, 2004):** A twice continuously differentiable payoff function \(\pi_i(x_1, x_2, \ldots, x_n)\) is supermodular if and only if \(\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} \geq 0\) for all \(x \neq i\) if its player’s payoffs are supermodular, the game is termed a supermodular game.

**Lemma 1 (Cachon and Netessine, 2004):** If a game is supermodular, at least one Nash equilibrium exists.

**Lemma 2 (Cachon and Netessine, 2004):** If an equilibrium solution exists and \(\sum_{i=1, i \neq k}^n \left| \frac{\partial^2 \pi_k}{\partial x_k \partial x_j} \right| > 0\), then the equilibrium solution is unique.

In Lemma 2, the right (left) side of the inequality can be interpreted as the effect of the firm \(i\)'s (firm \(j\)'s) decision on the best response of firm \(i\) (\(j\)). If firm \(i\) itself can act to counter firm \(j\)'s effect on its best response, the equilibrium is stable (Dai et al. 2005). The conditions, \(\sum_{i=1, i \neq k}^n \left| \frac{\partial^2 \pi_k}{\partial x_k \partial x_j} \right| > 0\), are defined as the “contraction mapping conditions” in the diagonal dominance form which has been extensively used by Bernstein and Federgruen (2004) and Dai et al. (2005). In the meanwhile, an iterative game could be viewed as a contraction mapping. According to the concept of stability of a Nash equilibrium studied by Moulin (1986), the outcome of an iterative game with changing first-movers will converge to an equilibrium solution, when the contraction mapping conditions hold. Following and applying the discussed concepts in Topkis (1978) and Moulin (1986), we proved the existence of Nash equilibrium and obtain the uniqueness condition for the model according to the Theorems 1 and 2.

**Theorem 1.** \(\pi_m, \pi_{m2}\) and \(\pi_r\) are supermodular in \((p_0, p_1, p_2, p_3)\).

We checked that conditions \(\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} \geq 0\) are always true for all \(x \neq i\) within the model.

Then, Lemma 1 confirmed the existence of Nash equilibrium.

By applying Lemma 2, we obtained sufficient conditions for the uniqueness of the Nash equilibrium in our model. These conditions can be stated as in the Theorem 2.

**Theorem 2.** The Nash equilibrium is unique if Equations (23)-(26) hold.

\[
\begin{align*}
(b_0 > (b_0 + \beta_1 + \alpha_2)/2) \\
(b_1 > (\theta_1 z/ (\beta_2 + \beta_0 + \alpha_1) + \beta_2 z_2) / 2 \theta_1 z_1) \\
(b_2 > (\theta_2 z/ (\beta_2 + \beta_1 + \alpha_0) + \beta_2 z_2) / 2 \theta_2 z_2) \\
(b_3 > (\alpha_1 + \alpha_2)/2)
\end{align*}
\]

We proved that corresponding conditions of Lemma 2 \(\sum_{i=1, i \neq k}^n \left| \frac{\partial^2 \pi_k}{\partial x_k \partial x_j} \right| > 0\) are true if Equations (23)-(26) hold.
The superscript *N is used to denote the optimal value for the Nash game. The optimal retail prices \( (p_0^N, p_1^N, p_2^N, p_3^N) \) for the Nash game are obtained by simultaneously solving the system of equations presented in Equation (27):

\[
\begin{align*}
\frac{\partial \pi_R}{\partial p_0} &= \frac{\partial \pi_R}{\partial p_1} = \frac{\partial \pi_R}{\partial p_2} = \frac{\partial \pi_R}{\partial p_3} = 0
\end{align*}
\]  

(Eq. 27)

A closed form expression for the optimal retail prices is not reported here because of its complexity. \( p_i^N (j=0,1,2,3) \), \( \pi_i^N m(t) \), \( \pi_i^N m=0 \) and \( \pi_i^N \), increase with \( \beta_i \) and \( \alpha_i \), while they decrease with \( b_i (i,j=0,1,2,3) \); thus, \( b_i \) has a negative effect while \( \beta_i \) and \( \alpha_i \) have positive effects on the profits. Manufacturers and the retailer may act to adjust marketing activities to increase their respective profits. They may either decrease \( b_i \) by increasing consumer brand loyalty for the sold products through the traditional channel, or increase \( \beta_i \) and \( \alpha_i \) by using marketing activities such as advertising, providing better sales explanations, and improving product representation. The retailer and manufacturers do not increase their channel loyalty because \( \beta_0, \alpha_0 \) are decreased by increasing consumer loyalty to a channel. This finding is similar to that of Kurata, Yao, and Liu (2007).

**Pricing policy under Stackelberg game**

In this section, we investigate two different scenarios of the Stackelberg game to find whether manufacturers and the retailer enjoy different bargaining powers within the supply chain or not. In Scenario 1, at first, the retailer announces \( p_1 \) and \( p_2 \) to maximize his \( \pi \), as the leader. Then, in response to \( p_1 \) and \( p_2 \), Manufacturers 1 and 2 select \( p_0 \) and \( p_3 \) to maximize \( \pi_m (i=1,2) \) as the followers. Contrarily, in Scenario 2, Manufacturers 1 and 2 begin with choosing \( p_0 \) and \( p_3 \) in such a way to maximize \( \pi_m (i=1,2) \), in response to which the retailer decides on \( p_1 \) and \( p_2 \) so as to maximize \( \pi \). It is necessary to derive the required conditions to guarantee the existence of a unique equilibrium solution for the Stackelberg game, similar to that of the Nash game. Superscripts \(^S_1\) and \(^S_2\) are used to denote the optimal values for Scenarios 1 and 2 of the Stackelberg game, respectively.

**Pricing policy under retailer-leader Stackelberg(RS) game (Scenario 1):**

In this scenario, the retailer has more bargaining power than the manufacturers and is the Stackelberg game leader. By setting \( \frac{\partial \pi_{m1}}{\partial p_0} = 0 \) and solving for \( p_0 \), and also by setting \( \frac{\partial \pi_{m2}}{\partial p_3} = 0 \) and solving for \( p_3 \), the optimal values of \( p_0 \) and \( p_3 \) can be obtained in terms of \( p_1 \) and \( p_2 \). Then, by substituting \( p_0 \) and \( p_3 \) into \( \pi_R \) and setting \( \frac{\partial \pi_R}{\partial p_1} = \frac{\partial \pi_R}{\partial p_2} = 0 \) and finally solving for \( p_1 \) and \( p_2 \), the retailer and the manufacturer’s profit-maximizing retail prices \( (p_0^{S1}, p_1^{S1}, p_2^{S1}, p_3^{S1}) \) for the RS game can be obtained. A closed form expression for the optimal retail prices in scenario 1 is not reported here because of its complexity.

Based on Zhao, Tang, and Wei (2012), we obtained the uniqueness conditions for Scenario 1 expressed as Theorem 3.

**Theorem 3.** In Scenario 1 of the Stackelberg game, the equilibrium solution is unique if Equations (28)-(30) hold.

\[
4\theta_1\theta_2\theta_3(-(b_1^2 + \frac{2b_0^2}{b_0}) \alpha_2 \beta_0^2 + \frac{2a_0 b_0 + \alpha_2 \beta_0}{b_0} (\beta_0 \alpha_2 + \alpha_i)) \times (-b_2 + \frac{2b_0^2}{b_0} + \frac{2a_0 b_0 + \alpha_2 \beta_0}{b_0} (\beta_0 \alpha_2 + \alpha_i)) 
\]

\[
(\theta_1\theta_2\theta_3 (\frac{b_1}{b_0} + \frac{2a_0 b_0 + \alpha_2 \beta_0}{b_0} (\beta_0 \alpha_2 + \alpha_i) + \beta_1 + \theta_2\theta_3 (\frac{b_0}{b_0} + \frac{2a_0 b_0 + \alpha_2 \beta_0}{b_0} (\beta_0 \alpha_2 + \alpha_i)) \beta_1) \times (\theta_1\theta_2\theta_3 (\frac{b_0}{b_0} + \frac{2a_0 b_0 + \alpha_2 \beta_0}{b_0} (\beta_0 \alpha_2 + \alpha_i) + \beta_1 + \theta_2\theta_3 (\frac{b_0}{b_0} + \frac{2a_0 b_0 + \alpha_2 \beta_0}{b_0} (\beta_0 \alpha_2 + \alpha_i)) \beta_1))^2
\]  

(Eq. 28)

\[
\theta_1\theta_2\theta_3 (-2b_1 + \frac{a_2}{2b_0}) < 0
\]  

(Eq. 29)
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\[ -2b_0\delta q_0\theta_0 \leq 0 \]

(Eq. 30)

3.2.2. Pricing policy under manufacturers-leader Stackelberg (MS) game (Scenario 2):

In this scenario, the manufacturers are the leaders holding more bargaining power than that of the retailer. By setting \( \partial \pi_R / \partial p_1, \partial \pi_R / \partial p_2 \) equal to zero and solving for \( p_1 \) and \( p_2 \), the optimal retail prices \( p_1 \) and \( p_2 \) can be obtained in terms of \( p_0 \) and \( p_3 \). By substituting \( p_1 \) and \( p_2 \) into \( \pi_{m1} \) and \( \pi_{m2} \), respectively, and also setting \( \partial \pi_{m1} / \partial p_0 = \partial \pi_{m2} / \partial p_3 = 0 \) and solving for \( p_0 \) and \( p_3 \), the optimal retail prices \( (p_0^{\ast}, p_1^{\ast}, p_2^{\ast}, p_3^{\ast}) \) are obtained for the MS game. A closed form expression for the optimal retail prices in Scenario 2 is not reported here because of its complexity.

We check uniqueness conditions for Scenario 2 in accordance with Zhao, Tang, and Wei (2012) (See Theorem 4).

Theorem 4. In Scenario 2 of the Stackelberg game, the equilibrium solution is unique if Equations (31)-(33) hold.

\[
40z_ib_1b_2z_1z_2b_2 \leq (b_2b_1z_1+b_2z_0b_2)^2 \] \hfill (Eq. 31)

\[
\{20z_0b_0(z_0b_0b_2(b_2z_0z_1+b_2z_0b_2)+2b_1b_2z_0b_2))((b_2^2 (z_0+b_2z_2z_0b_2)+40z_1z_1+b_2z_2z_0b_2)(b_2+b_2(b_2z_2z_0b_2))\} < 0 \] \hfill (Eq. 32)

\[
20z_1z_2(z_0b_0z_0z_2+b_1b_2z_0z_0)((-b_2^2 (z_0+b_2z_2z_0b_2)+40z_1z_1+b_2z_2z_0b_2)+\alpha_1(z_0+b_2z_2z_0b_2)+2b_1b_2z_0z_0b_2)\} < 0 \] \hfill (Eq. 33)

Numerical examples

In this section, we give more managerial insights using some numerical examples. To avoid the complexity and make comparable results, we assume identical values for the products’ parameters as those in Hsieh and Wu, 2009, and Kurata et al., 2007. Considering \( i=0,1,2,3 \) representing Product 1 through the internet channel, Product 1 through the traditional channel, Product 2 through the traditional channel, Product 2 through the internet channel, respectively, we assume identical values for the unit production costs (i.e. \( c_i=c \)), wholesale prices (i.e. \( w_i=w \)), deliverable quantity uncertainties (i.e. \( \gamma_i=\gamma \)), potential demands (i.e. \( \alpha_i=\alpha \)), service values (i.e. \( \gamma_i=\gamma \)), service cost factors (i.e. \( \eta_i=\eta \)), unit shortage costs (i.e. \( t_i=t \)), salvage values of unsold and unallocated inventories (i.e. \( m= m \), \( s=s \)) and stock factors (i.e. \( z=z \)). The given identical values for parameters make Products 1 and 2 completely substitutable. Furthermore, consumers are assumed to have identical price sensitivities (i.e. \( b_i=b, b_i=b_i=b_1=\alpha_0=\alpha_1=\alpha_2 \)). The values of parameters are considered as \( a=100, b=10, \beta=4, \alpha=4, m=0.5, s=0.5, t=0.5, c=6, w=9, v=3, z=1, \eta=0.5 \) taken from the numerical studies in Hsieh and Wu, 2009; Kurata et al., 2007. The random variables \( x \) and \( y \) are considered to be uniformly distributed over \([1-\bar{x},1+\bar{x}]\) and \([1-\bar{y},1+\bar{y}]\), respectively, where \( \bar{x} \) and \( \bar{y} \) are determined based on the following coefficients of variations: \( cv_x=0.2 \) and \( cv_y=0.35 \). Furthermore, substituting the given values of parameters in Equations (16)-(18), one may find \( \theta = 0.77, K=1 \) and \( \lambda = 0.82 \).

Effects of variations in price sensitivity parameters

In this section, the effects of variations in \( b, b_1 \) and \( b_2 \) are examined. Each parameter is assumed to vary by ±30%. Figures 2-10 indicate that self-price sensitivity has a negative effect on profit, thus if the retailer and manufacturers can reduce the consumer sensitivity to price by increasing brand and channel loyalty, they will gain more profit. The addressed figures indicate that brand and channel price sensitivity have positive effects on profit. Figures 2-10 indicate that the effect
of $\beta_2$ (the cross-brand price sensitivity between Products 1 and 2, both sold through the traditional channel) is stronger than that of other price sensitivity parameters; therefore, the retailer should increase consumer brand loyalty for Products 1 and 2 through such actions as improving product displays in the traditional channel and providing a comfortable shopping environment for consumers. This means that consumer brand loyalty is more critical than channel loyalty for both the manufacturers and the retailer.

Figure 2. Effects of parameters variations on retailer’s profit under Nash game.

Figure 3. Effects of parameters’ variations on Manufacturer 1’s profit under Nash game.
Figure 4. Effects parameters’ variations on manufacturer 2’s profit under Nash game.

Figure 5. Effects of parameters’ variations on retailer’s profit under RS game.

Figure 6. Effects of parameters’ variations on Manufacturer 1’s profit under RS game.
Figure 7. Effects of parameters’ variations on manufacturer 2’s profit under RS game.

Figure 8. Effects of parameters’ variations on retailer’s profit under MS game.

Figure 9. Effects of parameters’ variations on Manufacturer 1’s profit under MS game.
Effect of competition strategy

The profits gained by the manufacturers and the retailer are given in Table 2 for the three scenarios. According to this table, the manufacturers make more profit as the followers in Scenario 1, while the retailer makes more profit as the follower in Scenario 2. Hence, being a follower in a Stackelberg game is not always harmful.

Table 2. Optimal profits of the manufacturers and the retailer for different competition strategies.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\pi_{m1}^*$</th>
<th>$\pi_{m2}^*$</th>
<th>$\pi_r^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash game</td>
<td>3443</td>
<td>3443</td>
<td>4174</td>
</tr>
<tr>
<td>Scenario 1</td>
<td>7479</td>
<td>7479</td>
<td>5564</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>4325</td>
<td>4325</td>
<td>5904</td>
</tr>
</tbody>
</table>

Effect of service value

Service value may encourage consumers to purchase more products to an extent depending on the price. Hence, the retailer cannot increase service value indefinitely. Focusing on the retailer’s profit in Figure 11, one can see that the retailer’s profit increases with $v$ at the beginning, but as $v$ becomes larger than the critical value (approximately 2 in the given example), any further increase in $v$ causes a decrease in the retailer’s profit, because some consumers can no more afford high prices caused by increasing service values.

Figure 10. Effects of parameters’ variations on Manufacturer 2’s profit under MS game.

Figure 11. Effect of $v$ on retailer’s profits.
Therefore, the retailer can add an appropriate service value to the product to erode the Internet channel, attract more consumers, and improve the profit.

CONCLUSION

The main contribution of this paper is to formulate a pricing scheme for substitutable products in a two-echelon supply chain composed of two different manufacturers with their own Internet channels and one independent retailer. Considering a stochastic demand function for the consumers, three scenarios are developed based on Nash and Stackelberg game approaches in order to investigate pricing policies and uniqueness conditions for equilibrium solutions. Beside performing a numerical analysis, some managerial insights are given to the economic behavior of companies looking to gain more profits. The retailer can create more demand and gain more profit through providing better services, but the numerical findings show that the retailer cannot unlimitedly improve service values since from some point on, the corresponding service cost may suppress the demand. The results show that both manufacturers and the retailer can benefit from being the follower in the Stackelberg games and playing such a role is not necessarily useless. As the results of numerical analysis demonstrate, the manufacturers and the retailer prefer Stackelberg game over Nash game with more profit being gained by the followers. Finally, several ideas are proposed for future research as follows:

- Considering an Internet channel for the retailer.
- Considering the case where added service value and the wholesale price are not exogenous, rather they are taken as decision variables.
- Extending this single period model to a multi-period one.
- Considering the centralized supply chain and using coordination mechanisms such as revenue sharing.
- Considering return policy in order to decrease the effects of uncertainties on demand.

REFERENCES

Modeling and Solving A Pricing Problem Considering Substitutable Products In A Dual Supply Chain With Internet and Traditional Distribution Channels